RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2015

FIRST YEAR

STATISTICS (General)

Paper : II

Date : 27/05/2015

Time : 11 am – 1 pm

Full Marks : 50

(Use a separate Answer Book for each group)

Group – A

1. Answer **any two** questions of the following : (2 × 5)

   a) What do you mean by independence of attributes? Give a criterion of independence of attributes A & B.

   b) If $R_{1.23} = 1$, show that $R_{2.13} = R_{3.12} = 1$

   c) Define Yule's coefficient of association and the coefficient of colligation. How are they linked?

   d) Show that if $X_3 = aX_1 + bX_2$, then $r_{13.2}$ has a numerical value 1 having the same sign of a.

2. Answer **any one** question of the following : (1 × 15)

   a) i) Derive the linear regression equation of X_1 on X_2 & X_3. (8)

   ii) Show that multiple correlation coefficient $R_{1.23}$ can be expressed as : $(1 - R^2_{1.23}) = R / R_{11}$, where R is the determinant of the correlation matrix and R_{11} is the cofactor of r_{11} in R. (7)

   b) i) Suppose a computer has found for a given set of values of variables X_1, X_2 & X_3, the correlation coefficients as $r_{12} = 0.91$, $r_{13} = 0.33$ and $r_{23} = 0.81$. Check the results for any inconsistency. (5)

   ii) Consider the regression equation of first year marks (FYM), higher secondary marks (HS) and Madhyamik marks (MP) , obtained as FYM = -5 + 0.6 HS + 0.2 MP, with $R^2 = 0.35$ (all marks are in percentages and R denoting the multiple correlation coefficient)

   (I) What is the percentage of the variability in FYM not explained by HS and MP. Justify.

   (II) Soumya has score of MP which is higher than Anirban's by 10, but Anirban has a higher score in HS than Soumya's by 4. Who is expected to score higher in FYM? Justify. (5 + 5)

Group – B

3. Answer **any two** questions of the following : (2 × 5)

   a) i) A radio-active source emits on the average 2.5 particles / second. Find the probability that 3 or more particles will emit in an interval of 4 seconds. (3)

   ii) Let X ~ Binomial (n, p). For what value of p is the variance of X maximum? (2)

   b) Given below is the joint probability density function of two random variables X & Y :

   $f(x,y) = 3x(y+x)/5$ for $0 < x < 1$, $0 < y < 2$

   $= 0$; otherwise

   Find P [(x,y) ϵ A] , where A is the region { (x,y) | $0 <x<1/2$, $1 <y<2$} (5)

   c) If X has an exponential distribution, then show that $P [X >= t + T \mid X >= T] = P[X >=t]$ (5)

   d) Use the Normal approximation to the binomial distribution to determine the probability that number of heads lies between 6 & 8 in 16 flips of a balanced coin. (5)

4. Answer **any one** question of the following : (1 × 15)

a) i) State the central limit theorem. (2)

ii) Show, in the context of bivariate Normal distribution, the value of correlation coefficient as zero implies the independence of two random variables. (3)

iii) Prove the following result of conditional expectation :

$EE(Y|X) = E(Y)$, where X & Y are jointly distributed discrete random variables. (3)

iv) Suppose that during transcendental meditation, the reduction of a person's oxygen consumption is a random variable having a Normal distribution with mean 37.6 cc/minute & s.d = 4.6 cc/minute. Find the probabilities that during transcendental meditation, a person's oxygen consumption will be reduced by (7)

(I) at least 44.5 cc / minute.

(II) at most 35 cc / minute.

b) i) If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the 10^{th} child exposed to the disease will be the 3^{rd} to catch it? (3)

ii) Compute the r^{th} moment about origin of the Gamma distribution. From the expression, find the mean and variance of Gamma distribution. (4)

iii) Let $X \sim$ Poisson (λ). Show that, when λ is an integer, the mean and one of the modes of Poisson distribution are equal. (4)

iv) Find the value of 1^{st} quartile of the continuous uniform distribution whose pdf is as follows :

$f(x) = 1/2 \; ; \; 1 < x < 3$

$= 0$; Otherwise (4)

———— × ————

TABLE I Ordinates and Areas of the Distribution of Standard Normal Variable*

τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)
·00	·3989423	·5000000						
·01	·3989223	·5039894	·51	·3502919	·6949743	1·01	·2395511	·8437524
·02	·3988625	·5079783	·52	·3484925	·6984682	1·02	·2371320	·8461358
·03	·3987628	·5119665	·53	·3466677	·7019440	1·03	·2347138	·8484950
·04	·3986233	·5159534	·54	·3448180	·7054015	1·04	·2322970	·8508300
·05	·3984439	·5199388	·55	·3429439	·7088403	1·05	·2298821	·8531409
·06	·3982248	·5239222	·56	·3410458	·7122603	1·06	·2274696	·8554277
·07	·3979661	·5279032	·57	·3391243	·7156612	1·07	·2250599	·8576903
·08	·3976677	·5318814	·58	·3371799	·7190427	1·08	·2226535	·8599289
·09	·3973298	·5358564	·59	·3352132	·7224047	1·09	·2202508	·8621434
·10	·3969525	·5398278	·60	·3332246	·7257469	1·10	·2178522	·8643339
·11	·3965360	·5437953	·61	·3312147	·7290691	1·11	·2154582	·8665005
·12	·3960802	·5477584	·62	·3291840	·7323711	1·12	·2130691	·8686431
·13	·3955854	·5517168	·63	·3271330	·7356527	1·13	·2106856	·8707619
·14	·3950517	·5556700	·64	·3250623	·7389137	1·14	·2083078	·8728568
·15	·3944793	·5596177	·65	·3229724	·7421539	1·15	·2059363	·8749281
·16	·3938684	·5635595	·66	·3208638	·7453731	1·16	·2035714	·8769756
·17	·3932190	·5674949	·67	·3187371	·7485711	1·17	·2012135	·8789995
·18	·3925315	·5714237	·68	·3165929	·7517478	1·18	·1988631	·8809999
·19	·3918060	·5753454	·69	·3144317	·7549029	1·19	·1965205	·8829768
·20	·3910427	·5792597	·70	·3122539	·7580363	1·20	·1941861	·8849303
·21	·3902419	·5831662	·71	·3100603	·7611479	1·21	·1918602	·8863606
·22	·3894038	·5870644	·72	·3078513	·7642375	1·22	·1895432	·8887676
·23	·3885286	·5909541	·73	·3056274	·7673049	1·23	·1872354	·8906514
·24	·3876166	·5948349	·74	·3033893	·7703500	1·24	·1849373	·8925123
·25	·3866681	·5987063	·75	·3011374	·7733726	1·25	·1826491	·8943502
·26	·3856834	·6025681	·76	·2988724	·7763727	1·26	·1803712	·8961653
·27	·3846627	·6064199	·77	·2965948	·7793501	1·27	·1781038	·8979577
·28	·3836063	·6102612	·78	·2943050	·7823046	1·28	·1758474	·8997274
·29	·3825146	·6140919	·79	·2920038	·7852361	1·29	·1736022	·9014747
·30	·3813878	·6179114	·80	·2896916	·7881446	1·30	·1713686	·9031995
·31	·3802264	·6217195	·81	·2873689	·7910299	1·31	·1691468	·9049021
·32	·3790305	·6255158	·82	·2850364	·7938919	1·32	·1669370	·9065825
·33	·3778007	·6293000	·83	·2826945	·7967306	1·33	·1647397	·9082409
·34	·3765372	·6330717	·84	·2803438	·7995458	1·34	·1625551	·9098773
·35	·3752403	·6368307	·85	·2779849	·8023375	1·35	·1603833	·9114920
·36	·3739106	·6405764	·86	·2756182	·8051055	1·36	·1582248	·9130850
·37	·3725483	·6443088	·87	·2732444	·8078498	1·37	·1560797	·9146565
·38	·3711539	·6480273	·88	·2708640	·8105703	1·38	·1539483	·9162067
·39	·3697277	·6517317	·89	·2684774	·8132671	1·39	·1518308	·9177356
·40	·3682701	·6554217	·90	·2660852	·8159399	1·40	·1497275	·9192433
·41	·3667817	·6590970	·91	·2636880	·8185887	1·41	·1476385	·9207302
·42	·3652627	·6627573	·92	·2612863	·8212136	1·42	·1455641	·9221962
·43	·3637136	·6664022	·93	·2588805	·8238145	1·43	·1435046	·9236415
·44	·3621349	·6700314	·94	·2564713	·8263912	1·44	·1414600	·9250663
·45	·3605270	·6736448	·95	·2540591	·8289439	1·45	·1394306	·9264707
·46	·3588903	·6772419	·96	·2516443	·8314724	1·46	·1374165	·9278550
·47	·3572253	·6808225	·97	·2492277	·8339768	1·47	·1354181	·9292191
·48	·3555325	·6843863	·98	·2468095	·8364569	1·48	·1334353	·9305634
·49	·3538124	·6879331	·99	·2443904	·8389129	1·49	·1314684	·9318879
·50	·3520653	·6914625	1·00	·2419707	·8413447	1·50	·1295176	·9331928

TABLE I (*Contd.*)

τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)
1·51	·1275830	·9344783	2·01	·0529192	·9777844	2·51	·0170947	·9939634
1·52	·1256646	·9357445	2·02	·0518636	·9783083	2·52	·0166701	·9941323
1·53	·1237628	·9369916	2·03	·0508239	·9788217	2·53	·0162545	·9942969
1·54	·1218775	·9382198	2·04	·0498001	·9793248	2·54	·0158476	·9944574
1·55	·1200090	·9394292	2·05	·0487920	·9798178	2·55	·0154493	·9946139
1·56	·1181573	·9406201	2·06	·0477996	·9803007	2·56	·0150596	·9947664
1·57	·1163225	·9417924	2·07	·0468226	·9807738	2·57	·0146782	·9949151
1·58	·1145048	·9429466	2·08	·0458611	·9812372	2·58	·0143051	·9950600
1·59	·1127042	·9440826	2·09	·0449148	·9816911	2·59	·0139401	·9952012
1·60	·1109208	·9452007	2·10	·0439836	·9821356	2·60	·0135830	·9953388
1·61	·1091548	·9463011	2·11	·0430674	·9825708	2·61	·0132337	·9954729
1·62	·1074061	·9473839	2·12	·0421661	·9829970	2·62	·0128921	·9956035
1·63	·1056748	·9484493	2·13	·0412795	·9834142	2·63	·0125581	·9957308
1·64	·1039611	·9494974	2·14	·0404076	·9838226	2·64	·0122315	·9958547
1·65	·1022649	·9505285	2·15	·0395500	·9842224	2·65	·0119122	·9959754
1·66	·1005864	·9515428	2·16	·0387069	·9846137	2·66	·0116001	·9960930
1·67	·0989255	·9525403	2·17	·0378779	·9849966	2·67	·0112951	·9962074
1·68	·0972823	·9535213	2·18	·0370629	·9853713	2·68	·0109969	·9963189
1·69	·0956568	·9544860	2·19	·0362619	·9857379	2·69	·0107056	·9964274
1·70	·0940491	·9554345	2·20	·0354746	·9860966	2·70	·0104209	·9965330
1·71	·0924591	·9563671	2·21	·0347009	·9864474	2·71	·0101428	·9966358
1·72	·0908870	·9572838	2·22	·0339408	·9867906	2·72	·0098712	·9967359
1·73	·0893326	·9581849	2·23	·0331939	·9871263	2·73	·0096058	·9968333
1·74	·0877961	·9590705	2·24	·0324603	·9874545	2·74	·0093466	·9969280
1·75	·0862773	·9599408	2·25	·0317397	·9877755	2·75	·0090936	·9970202
1·76	·0847764	·9607961	2·26	·0310319	·9880894	2·76	·0088465	·9971099
1·77	·0832932	·9616364	2·27	·0303370	·9883962	2·77	·0086052	·9971972
1·78	·0818278	·9624620	2·28	·0296546	·9886962	2·78	·0083697	·9972821
1·79	·0803801	·9632730	2·29	·0289847	·9889893	2·79	·0081398	·9973646
1·80	·0789502	·9640697	2·30	·0283270	·9892759	2·80	·0079155	·9974449
1·81	·0775379	·9648521	2·31	·0276816	·9895559	2·81	·0076965	·9975229
1·82	·0761433	·9656205	2·32	·0270481	·9898296	2·82	·0074829	·9975988
1·83	·0747663	·9663750	2·33	·0264265	·9900969	2·83	·0072744	·9976726
1·84	·0734068	·9671159	2·34	·0258166	·9903581	2·84	·0070711	·9977443
1·85	·0720649	·9678432	2·35	·0252182	·9906133	2·85	·0068728	·9978140
1·86	·0707404	·9685572	2·36	·0246313	·9908625	2·86	·0066793	·9978818
1·87	·0694333	·9692581	2·37	·0240556	·9911060	2·87	·0064907	·9979476
1·88	·0681436	·9699460	2·38	·0234910	·9913437	2·88	·0063067	·9980116
1·89	·0668711	·9706210	2·39	·0229374	·9915758	2·89	·0061274	·9980738
1·90	·0656158	·9712834	2·40	·0223945	·9918025	2·90	·0059525	·9981342
1·91	·0643777	·9719334	2·41	·0218624	·9920237	2·91	·0057821	·9981929
1·92	·0631566	·9725711	2·42	·0213407	·9922397	2·92	·0056160	·9982498
1·93	·0619524	·9731966	2·43	·0208294	·9924506	2·93	·0054541	·9983052
1·94	·0607652	·9738102	2·44	·0203284	·9926564	2·94	·0052963	·9983589
1·95	·0595947	·9744119	2·45	·0198374	·9928572	2·95	·0051426	·9984111
1·96	·0584409	·9750021	2·46	·0193563	·9930531	2·96	·0049929	·9984618
1·97	·0573038	·9755808	2·47	·0188850	·9932443	2·97	·0048470	·9985110
1·98	·0561831	·9761482	2·48	·0184233	·9934309	2·98	·0047050	·9985588
1·99	·0550789	·9767045	2·49	·0179711	·9936128	2·99	·0045666	·9986051
2·00	·0539910	·9772499	2·50	·0175283	·9937903	3·00	·0044318	·9986501

TABLE I (*Contd.*)

τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$
3·01	·0043007	·9986938	3·21	·0023089	·9993363	3·41	·0011910	·9996752
3·02	·0041729	·9987361	3·22	·0022358	·9993590	3·42	·0011510	·9996869
3·03	·0040486	·9987772	3·23	·0021649	·9993810	3·43	·0011122	·9996982
3·04	·0039276	·9988171	3·24	·0020960	·9994024	3·44	·0010747	·9997091
3·05	·0038098	·9988558	3·25	·0020290	·9994230	3·45	·0010383	·9997197
3·06	·0036951	·9988933	3·26	·0019641	·9994429	3·46	·0010030	·9997299
3·07	·0035836	·9989297	3·27	·0019010	·9994623	3·47	·0009689	·9997398
3·08	·0034751	·9989650	3·28	·0018397	·9994810	3·48	·0009358	·9997493
3·09	·0033695	·9989992	3·29	·0017803	·9994991	3·49	·0009037	·9997585
3·10	·0032668	·9990324	3·30	·0017226	·9995166	3·50	·0008727	·9997674
3·11	·0031669	·9990646	3·31	·0016666	·9995335	3·51	·0008426	·9997759
3·12	·0030698	·9990957	3·32	·0016122	·9995499	3·52	·0008135	·9997842
3·13	·0029754	·9991260	3·33	·0015595	·9995658	3·53	·0007853	·9997922
3·14	·0028835	·9991553	3·34	·0015084	·9995811	3·54	·0007581	·9997999
3·15	·0027943	·9991836	3·35	·0014587	·9995959	3·55	·0007317	·9998074
3·16	·0027075	·9992112	3·36	·0014106	·9996103	3·56	·0007001	·9998146
3·17	·0026231	·9992378	3·37	·0013639	·9996242	3·57	·0006814	·9998215
3·18	·0025412	·9992636	3·38	·0013187	·9996376	3·58	·0006575	·9998282
3·19	·0024615	·9992886	3·39	·0012748	·9996505	3·59	·0006343	·9998347
3·20	·0023841	·9993129	3·40	·0012322	·9996631	3·60	·0006119	·9998409

*Abridged from Table 1 of *Biometrika Tables for Statisticians*, vol. I, with the kind permission of the Biometrika Trustees.

TABLE II STANDARD NORMAL DISTRIBUTION

Values of τ_α

α	0·05	0·025	0·01	0·005
τ_α	1·645	1·960	2·326	2·576